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Neutrino oscillations in a supersymmetric SO(10) model with type-III see-saw mechanism

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Abstract: The neutrino oscillations are studied in the framework of the minimal supersymmetric SO(10) model with type-III see-saw mechanism by additionally introducing a number of SO(10) singlet neutrinos. The light Majorana neutrino mass matrix is given by a combination of those of the singlet neutrinos and the $SU(2)_L$ active neutrinos. The minimal SO(10) model gives an unambiguous Dirac neutrino mass matrix, which enables us to predict the masses and the other parameters for the singlet neutrinos. These predicted masses take the values accessible and testable by near future collider experiments under the reasonable assumptions. More comprehensive calculations on these parameters are also given.

Keywords: Neutrino Physics, Beyond Standard Model, GUT.

JHEP06(2005)016

Contents

1. Introduction

As pointed out in [1], we can construct, within the context of the standard model (SM), an operator which gives rise to the neutrino masses as

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda}(\ell_L H)^T C^{-1}(\ell_L H). \tag{1.1}$$

Here ℓ, H are the lepton doublet and the Higgs doublet, C is the charge conjugation operator and Λ is the scale in which something new physics appears. In the usual see-saw mechanism (type-I see-saw mechanism) [2], the scale parameter Λ is interpreted as the energy scale at which the right-handed neutrinos become active. In this paper, we explore the other possibility of type-III seesaw, introducing a set of singlet into the minimal supersymmetric standard model (MSSM). The motivations of this are as follows. One comes from the theoretical reason that string inspired E_6 models include SO(10) singlets as a matter content. The other does from the empirical reason that many indicate reduced coupling of neutrinos to the Z^0-boson in the framework of the SM or the SM with right-handed neutrinos [3, 4].

2. Type-III see-saw mechanism

We begin with reviewing the essential concept of the type-III see-saw mechanism proposed in the reference [5−9]. You can find a detailed study in [10]. In this model, in addition to the usual $SU(2)_L$ singlet $N = \nu_R$, we add a new SO(10) singlet neutrino "S", which has a positive lepton number (+1),

$$\mathcal{L}_Y = \int d^2\theta \left(Y_\nu\, \overline{N}\ell_L\, H_u + Y_s\, \overline{N} S_L\, H_s + \mu_s\, S_L^T C^{-1} S_L\right) + \text{h.c.}, \tag{2.1}$$

where H_u and H_s are the $SU(2)_L$ doublet and singlet chiral superfields, respectively. This lagrangian is written in a matrix form in the base with $\{\nu_L,\ N,\ S_L\}$ as follows:

$$\begin{pmatrix} 0 & m_D^T & 0 \\ m_D & 0 & M_D^T \\ 0 & M_D & \mu_s \end{pmatrix}. \tag{2.2}$$

After the spontaneous symmetry breaking, they give masses to the neutrinos as

$$m_D = Y_\nu \left\langle H_u^0 \right\rangle , \qquad M_D = Y_s \left\langle H_s \right\rangle . \tag{2.3}$$

Note that the μ_s term in the above breaks an originally existing global $\mathrm{U}(1)_L$ and $\mathrm{U}(1)_{\mathcal{R}}$ symmetries. Thus we can naturally expect it as a small value compared with the electroweak scale even around the keV scale, according to the following reason: when the μ_s term is arisen from the VEV of a singlet $\mu_s = \lambda \langle S' \rangle$, there appears a pseudo-NG boson, called Majoron $J = \Im S'$ associated with the spontaneously broken $\mathrm{U}(1)_L$ symmetry. Then the keV scale lepton number violation may lead to an interesting signature in the neutrinoless double beta decay [11] or becomes a possible candidate for the cold dark matter [12].

By integrating out ν_R,

$$\frac{\partial \mathcal{L}}{\partial N} = m_D \nu_L + M_D S_L = 0 , \tag{2.4}$$

we obtain

$$S_L = - \left(\frac{m_D}{M_D} \right) \nu_L . \tag{2.5}$$

This means that the light neutrino mass eigenstate is a linear combination of two states ν_L and S_L with the mixing angle $\epsilon \cong m_D / M_D$:

$$\nu_{\text{light}} = \nu_L - \epsilon \ S_L . \tag{2.6}$$

Such an extra mixing term is interesting when we try to explain the "NuTeV anomaly" through the heavy singlet neutrino contributions to the neutrino–nucleon scatterings [3, 4].

Putting eq. (2.5) into eq. (2.1), we get the effective light neutrino mass matrix as

$$M_\nu = \mu_s \left(\frac{m_D^T m_D}{M_D^2} \right) . \tag{2.7}$$

In general, adding three singlet neutrinos $\{S_1, S_2, S_3\}$, the effective light neutrino mass matrix can be written in the matrix form as

$$M_\nu = \left(M_D^{-1} m_D \right)^T \mu_s \left(M_D^{-1} m_D \right) . \tag{2.8}$$

This matrix is diagonalised by Maki-Nakagawa-Sakata (MNS) mixing matrix U as

$$U^T M_\nu \ U = \mathrm{diag}(m_1, m_2, m_3) . \tag{2.9}$$

An important fact is that the new physics scale has also the "see-saw structure" as

$$\Lambda \cong \frac{M_D^2}{\mu_s} . \tag{2.10}$$

Hence this mechanism is sometimes called as "double see-saw" mechanism. It's not the actual see-saw type but the inverse see-saw form, because the small lepton number violating ($\not{L}$) scale μ_s would indicate the large scale.

Now we consider the general three generation cases. For simplicity, we assume that all S_i have a common μ_s term. Then the light neutrino matrix is written as

$$\mu_s\, U^T m_D^T (M_D M_D^T)^{-1} m_D U = \mathrm{diag}(m_1, m_2, m_3)\,, \tag{2.11}$$

that is,

$$M_D M_D^T = m_D U \mathrm{diag}\left(\frac{\mu_s}{m_1}, \frac{\mu_s}{m_2}, \frac{\mu_s}{m_3}\right) U^T m_D^T\,. \tag{2.12}$$

This symmetric combination can be diagonalised by a single unitary matrix $\mathcal{U}$

$$\mathcal{U}^T\; M_D M_D^T\, \mathcal{U} = \mathrm{diag}(M_{D1}^2, M_{D2}^2, M_{D3}^2)\,. \tag{2.13}$$

Here we note that $\mathcal{U}$ includes three mixing angles θ_1', θ_2', θ_3' and six phases (δ, ζ_2^L, ζ_3^L, ζ_1^R, ζ_2^R, ζ_3^R)

$$\mathcal{U} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\zeta_2^L} & 0 \\ 0 & 0 & e^{i\zeta_3^L} \end{pmatrix} \begin{pmatrix} c_3 c_1 & c_3 s_1 & s_3 e^{-i\delta} \\ -c_2 s_1 - s_2 c_1 s_3 e^{i\delta} & c_2 c_1 - s_2 s_1 s_3 e^{i\delta} & s_2 c_3 \\ s_2 s_1 - c_2 c_1 s_3 e^{i\delta} & -s_2 c_1 - c_2 s_1 s_3 e^{i\delta} & c_2 c_3 \end{pmatrix} \times$$
$$\times \begin{pmatrix} e^{i\zeta_1^R} & 0 & 0 \\ 0 & e^{i\zeta_2^R} & 0 \\ 0 & 0 & e^{i\zeta_3^R} \end{pmatrix}, \tag{2.14}$$

where $s_i := \sin\theta_i'$, $c_i := \cos\theta_i'$. You should not confuse these mixing angles with those of the MNS mixing matrix U appearing in eq. (3.15).

From this expression, we can obtain a prediction about masses and mixings for the heavier Dirac mass matrix M_D by giving some informations about the light neutrino masses and mixings and the lighter Dirac mass matrix m_D.

3. Fermion masses in an SO(10) model with a singlet

In order to make a prediction on the second Dirac neutrino mass matrix M_D, we need an information for the Yukawa couplings of Y_ν. In this paper, we make the minimal SO(10) model extend to add a number of singlet, which preserves a precise information for m_D. We begin with a review of the minimal SUSY SO(10) model proposed in [13] and recently analysed in detail in references [14−21]. Even when we concentrate our discussion on the issue of how to reproduce the realistic fermion mass matrices in the SO(10) model, there are lots of possibilities of the introduction of Higgs multiplets. The minimal supersymmetric SO(10) model includes only one **10** and one $\overline{\mathbf{126}}$ Higgs multiplets in Yukawa couplings with **16** matter multiplets. Here, in addition to it, we introduce a number of SO(10) singlet chiral superfields **1** as new matter multiplets.[1] This additional singlet can provide a type-III see-saw mechanism as described in the previous section. In order to avoid a large triplet VEV for $\overline{\mathbf{126}}_H$ unnecessary in type-III see-saw model, we use a $\mathrm{U}(1)_\mathcal{R}$ symmetry. The corresponding $\mathrm{U}(1)_\mathcal{R}$ charges are listed in table 1. Then the relevant superpotential can be written as

[1] The singlet matter multiplet may have it's origin in some E_6 representations **27** or **78** which are decomposed under the SO(10) subgroup as $\mathbf{27} = \mathbf{16} + \mathbf{10} + \mathbf{1}$, $\mathbf{78} = \mathbf{45} + \mathbf{16} + \overline{\mathbf{16}} + \mathbf{1}$. In such a case, the superpotential given in eq. (3.1) may be generated from the following E_6 invariant superpotential: $W_Y = Y_1^{ij}\mathbf{27}_i\mathbf{27}_j\mathbf{27}_H + Y_2^{ij}\mathbf{27}_i\mathbf{27}_j\mathbf{351}'_H + Y_3^{ij}\mathbf{27}_i\mathbf{78}_j\overline{\mathbf{27}}_H$.

fields	$\mathrm{U}(1)_{\mathcal{R}}$ charges
$\mathbf{16}_i$	-1
$\mathbf{10}_H$	$+4$
$\overline{\mathbf{126}}_H$	$+4$
$\overline{\mathbf{16}}_H$	$+2$
$\mathbf{16}_H$	-2
$\mathbf{1}_i$	$+1$

Table 1: $\mathrm{U}(1)_{\mathcal{R}}$ charges of the fields relevant for the quark and lepton mass matrices ($R[W] = +2$).

$$W_Y = Y_{10}^{ij} \mathbf{16}_i \mathbf{16}_j \mathbf{10}_H + Y_{126}^{ij} \mathbf{16}_i \mathbf{16}_j \overline{\mathbf{126}}_H + Y_s^{ij} \mathbf{16}_i \mathbf{1}_j \overline{\mathbf{16}}_H + \mu_s \mathbf{1}_i^2 \,. \tag{3.1}$$

At low energy after the GUT symmetry breaking, the superpotential leads to

$$\begin{aligned} W \,=\, & \left(Y_{10}^{ij} H_{10}^u + Y_{126}^{ij} H_{126}^u\right) u_i^c q_j + \left(Y_{10}^{ij} H_{10}^d + Y_{126}^{ij} H_{126}^d\right) d_i^c q_j + \\ & + \left(Y_{10}^{ij} H_{10}^u - 3Y_{126}^{ij} H_{126}^u\right) N_i \ell_j + \left(Y_{10}^{ij} H_{10}^d - 3Y_{126}^{ij} H_{126}^d\right) e_i^c \ell_j + \\ & + Y_s^{ij} N_i S_j H_s + \mu_s S_i^2 \,, \end{aligned} \tag{3.2}$$

where H_{10} and H_{126} correspond to the Higgs doublets in $\mathbf{10}_H$ and $\overline{\mathbf{126}}_H$. That is, we have two pairs of Higgs doublets. In order to keep the successful gauge coupling unification, we suppose that one pair of Higgs doublets (a linear combination of $H_{10}^{u,d}$ and $H_{126}^{u,d}$) is light while the other pair is heavy ($\simeq M_{\mathrm{GUT}}$). The light Higgs doublets are identified as the MSSM Higgs doublets (H_u and H_d) and given by

$$H_u = \widetilde{\alpha}_u \; H_{10}^u + \widetilde{\beta}_u \; H_{126}^u \,; \qquad H_d = \widetilde{\alpha}_d \; H_{10}^d + \widetilde{\beta}_d \; H_{126}^d \,, \tag{3.3}$$

where $\widetilde{\alpha}_{u,d}$ and $\widetilde{\beta}_{u,d}$ denote elements of the unitary matrix which rotate the flavour basis in the original model into the SUSY mass eigenstates. Omitting the heavy Higgs mass eigenstates, the low energy superpotential is described by only the light Higgs doublets H_u and H_d such that

$$\begin{aligned} W_Y \,=\, & \left(\alpha^u Y_{10}^{ij} + \beta^u Y_{126}^{ij}\right) u_i^c q_j H_u + \left(\alpha^d Y_{10}^{ij} + \beta^d Y_{126}^{ij}\right) d_i^c q_j H_d + \\ & + \left(\alpha^u Y_{10}^{ij} - 3\beta^u Y_{126}^{ij}\right) N_i \ell_j H_u \;+\; \left(\alpha^d Y_{10}^{ij} - 3\beta^d Y_{126}^{ij}\right) e_i^c \ell_j H_d + \\ & + Y_s^{ij} N_i S_j H_s + \mu_s S_i^2 \,, \end{aligned} \tag{3.4}$$

where the formulas of the inverse unitary transformation of eq. (3.3), $H_{10}^{u,d} = \alpha^{u,d} H_{u,d} + \cdots$ and $H_{126}^{u,d} = \beta^{u,d} H_{u,d} + \cdots$, have been used. Providing the Higgs VEV's, $\langle H_u \rangle = v \sin\beta$ and $\langle H_d \rangle = v \cos\beta$ with $v \simeq 174$ (GeV), the Dirac mass matrices can be read off as

$$\begin{aligned} M_u &= c_{10} M_{10} + c_{126} M_{126} \,, \\ M_d &= M_{10} + M_{126} \,, \\ m_D &= c_{10} M_{10} - 3 c_{126} M_{126} \,, \\ M_e &= M_{10} - 3 M_{126} \,, \end{aligned} \tag{3.5}$$

JHEP06(2005)016

where M_u, M_d, m_D and M_e denote up-type quark, down-type quark, Dirac neutrino and charged-lepton mass matrices, respectively. Note that all the quark and lepton mass matrices are characterised by only two basic mass matrices, M_{10} and M_{126}, and four complex coefficients c_{10} and c_{126}. In addition to the above mass matrices the above model indicates the mass matrices,

$$\begin{aligned} M_R &= c_R\, M_{126}\,, \\ M_L &= c_L\, M_{126}\,, \end{aligned} \tag{3.6}$$

together with M_D given in eq. (2.3). c_R and c_L correspond to the VEV's of $(\mathbf{10}, \mathbf{1}, \mathbf{3}) \subset \overline{\mathbf{126}}$ and $(\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1}) \subset \overline{\mathbf{126}}$, respectively [22]. If M_R, M_L, M_D terms dominate, they are called type-I, type-II, and type-III see-saw, respectively. In this paper, we consider the case $c_R = c_L = 0$, type-III. Here $c_R = 0$ means that the theory does not pass the Pati-Salam phase and is broken to the standard model directly.

The mass matrix formulas in eq. (3.5) leads to the GUT relation among the quark and lepton mass matrices,

$$M_e = c_d\,(M_d + \kappa M_u)\,, \tag{3.7}$$

where

$$c_d = -\frac{3c_{10} + c_{126}}{c_{10} - c_{126}}\,, \tag{3.8}$$

$$\kappa = -\frac{4}{3c_{10} + c_{126}}\,. \tag{3.9}$$

Without loss of generality, we can take the basis where M_u is real and diagonal, $M_u = D_u$. Since M_d is the symmetric matrix, it is described as $M_d = V_{\rm CKM}^*\, D_d\, V_{\rm CKM}^\dagger$ by using the CKM matrix $V_{\rm CKM}$ and the real diagonal mass matrix D_d. Considering the basis-independent quantities, $\mathrm{tr}[M_e^\dagger M_e]$, $\mathrm{tr}[(M_e^\dagger M_e)^2]$ and $\det[M_e^\dagger M_e]$, and eliminating $|c_d|$, we obtain two independent equations,

$$\left(\frac{\mathrm{tr}\left[\widetilde{M_e}^\dagger \widetilde{M_e}\right]}{m_e^2 + m_\mu^2 + m_\tau^2}\right)^2 = \frac{\mathrm{tr}\left[\left(\widetilde{M_e}^\dagger \widetilde{M_e}\right)^2\right]}{m_e^4 + m_\mu^4 + m_\tau^4}\,, \tag{3.10}$$

$$\left(\frac{\mathrm{tr}\left[\widetilde{M_e}^\dagger \widetilde{M_e}\right]}{m_e^2 + m_\mu^2 + m_\tau^2}\right)^3 = \frac{\det\left[\widetilde{M_e}^\dagger \widetilde{M_e}\right]}{m_e^2\, m_\mu^2\, m_\tau^2}, \tag{3.11}$$

where $\widetilde{M_e} \equiv V_{\rm CKM}^*\, D_d\, V_{\rm CKM}^\dagger + \kappa D_u$. With input data of six quark masses, three angles and one CP-phase in the CKM matrix and three charged-lepton masses, we can solve the above equations and determine κ and $|c_d|$, but one parameter, the phase of c_d, is left undetermined [14 − 16]. With input data of six quark masses, three angles and one CP-phase in the CKM matrix and three charged lepton masses, we solve the above equations and determine κ. The original basic mass matrices, M_{10} and M_{126}, are described by

$$
M_{10} = \frac{3+|c_d|e^{i\sigma}}{4} V_{\text{CKM}}^* \, D_d \, V_{\text{CKM}}^\dagger + \frac{|c_d|e^{i\sigma}\kappa}{4} D_u \,, \tag{3.12}
$$

$$
M_{126} = \frac{1-|c_d|e^{i\sigma}}{4} V_{\text{CKM}}^* \, D_d \, V_{\text{CKM}}^\dagger - \frac{|c_d|e^{i\sigma}\kappa}{4} D_u \,, \tag{3.13}
$$

as the functions of σ, the phase of c_d, with the solutions $|c_d|$ and κ determined by the GUT relation.

Now let us solve the GUT relation and determine $|c_d|$ and κ. Since the GUT relation of eq. (3.7) is valid only at the GUT scale, we first evolve the data at the weak scale to the corresponding quantities at the GUT scale with given $\tan\beta$ according to the renormalization group equations (RGE's) and use them as input data at the GUT scale. Note that it is non-trivial to find the solution of the GUT relation since the number of the free parameters (fourteen) is almost the same as the number of inputs (thirteen). The solution of the GUT relation exists only if we take appropriate input parameters. Taking the experimental data at the M_Z scale [23], we get the following values for charged fermion masses and the CKM matrix at the GUT scale, M_{GUT} with $\tan\beta = 10$:

$$
\begin{aligned}
m_u &= 0.000980\,, & m_c &= 0.285\,, & m_t &= 113\,, \\
m_d &= 0.00135\,, & m_s &= 0.0201\,, & m_b &= 0.996\,, \\
m_e &= 0.000326\,, & m_\mu &= 0.0687\,, & m_\tau &= 1.17\,,
\end{aligned}
$$

and

$$
V_{\text{CKM}}(M_G) = \begin{pmatrix} 0.975 & 0.222 & -0.000940 - 0.00289i \\ -0.222 - 0.000129i & 0.974 + 0.000124i & 0.0347 \\ 0.00864 - 0.00282i & -0.0337 - 0.000647i & 0.999 \end{pmatrix}
$$

in the standard parameterisation. The signs of the input fermion masses have been chosen to be $(m_u, m_c, m_t) = (+,-,+)$ and $(m_d, m_s, m_b) = (-,-,+)$. By using these outputs at the GUT scale as input parameters, we can solve eqs. (3.10) and (3.11) and find a solution:

$$
\begin{aligned}
\kappa &= -0.0103 + 0.000606i\,, \\
|c_d| &= 6.32\,.
\end{aligned} \tag{3.14}
$$

Once these parameters, $|c_d|$ and κ, are determined, we can describe all the fermion mass matrices as a functions of σ from the mass matrix formulas of eqs. (3.5), (3.12) and (3.13). Thus in the minimal SO(10) model we have almost unambiguous Dirac neutrino mass matrix m_D and, therefore, we can obtain the informations on M_D from the neutrino experiments via $M_\nu = (M_D^{-1} m_D)^T \mu_s (M_D^{-1} m_D)$ as in eq. (2.8).

Now we proceed to the numerical calculation of M_D from the well-confirmed neutrino oscillation data. The MNS mixing matrix U in the standard parametrization is

$$
U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12}e^{i\varphi_2} & s_{13}e^{i(\varphi_1-\delta)} \\ (-c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta})e^{-i\varphi_2} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13}e^{i(\varphi_1-\varphi_2)} \\ (s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta})e^{-i\varphi_1} & (-s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta})e^{-i(\varphi_1-\varphi_2)} & c_{23}c_{13} \end{pmatrix}, \tag{3.15}
$$

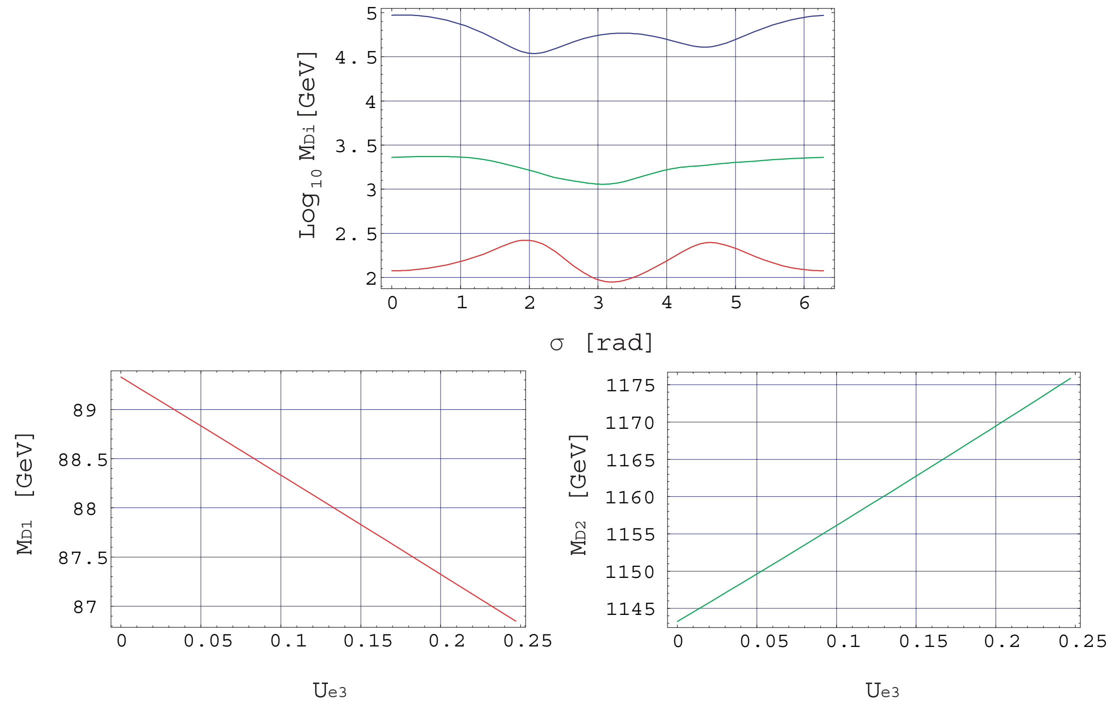

Figure 1: The predicted mass spectra of an additional singlet neutrino M_{Di} $(i = 1-3)$. The top panel represents three mass eigenvalues as a function of σ, the second and the third panels are the lightest and the second lightest masses as a function of U_{e3} for fixed $\sigma = \pi$.

where $s_{ij} := \sin\theta_{ij}$, $c_{ij} := \cos\theta_{ij}$ and δ, φ_1, φ_2 are the Dirac phase and the Majorana phases, respectively. Recent KamLAND data tells us that[2]

$$\begin{aligned}
\Delta m_{\oplus}^2 &= \Delta m_{32}^2 \;=\; 2.1 \times 10^{-3}\,\mathrm{eV}^2\,, \\
\sin^2\theta_{\oplus} &= 0.5\,, \\
\Delta m_{\odot}^2 &= \left|\Delta m_{21}^2\right| \;=\; 8.3 \times 10^{-5}\,\mathrm{eV}^2\,, \\
\sin^2\theta_{\odot} &= 0.28\,, \\
|U_{e3}|^2 &< 0.061\,. \qquad (3.16)
\end{aligned}$$

For simplicity we take $U_{e3} = 0$. Note that we can take both signs of Δm_{21}^2, $\Delta m_{21}^2 > 0$ or $\Delta m_{21}^2 < 0$. The former is called normal hierarchy, the latter is called inverted hierarchy. Here we adopt the former case, and take the lightest neutrino mass eigenvalue as $m_\ell = 10^{-3}$ (eV). Then the mass eigenvalues are written as

$$\begin{aligned}
m_1 &= m_\ell\,, \\
m_2 &= \sqrt{m_\ell^2 + \Delta m_{\oplus}^2}\,, \\
m_3 &= \sqrt{m_\ell^2 + \Delta m_{\oplus}^2 + \Delta m_{\odot}^2}\,. \qquad (3.17)
\end{aligned}$$

For the light Dirac neutrino mass matrix m_D, we input the SO(10) predicted one as was done in the previous section. However, unlike the case of minimal SO(10) GUT model, we

[2]Our convention is $\Delta m_{ij}^2 = m_i^2 - m_j^2$.

JHEP06(2005)016

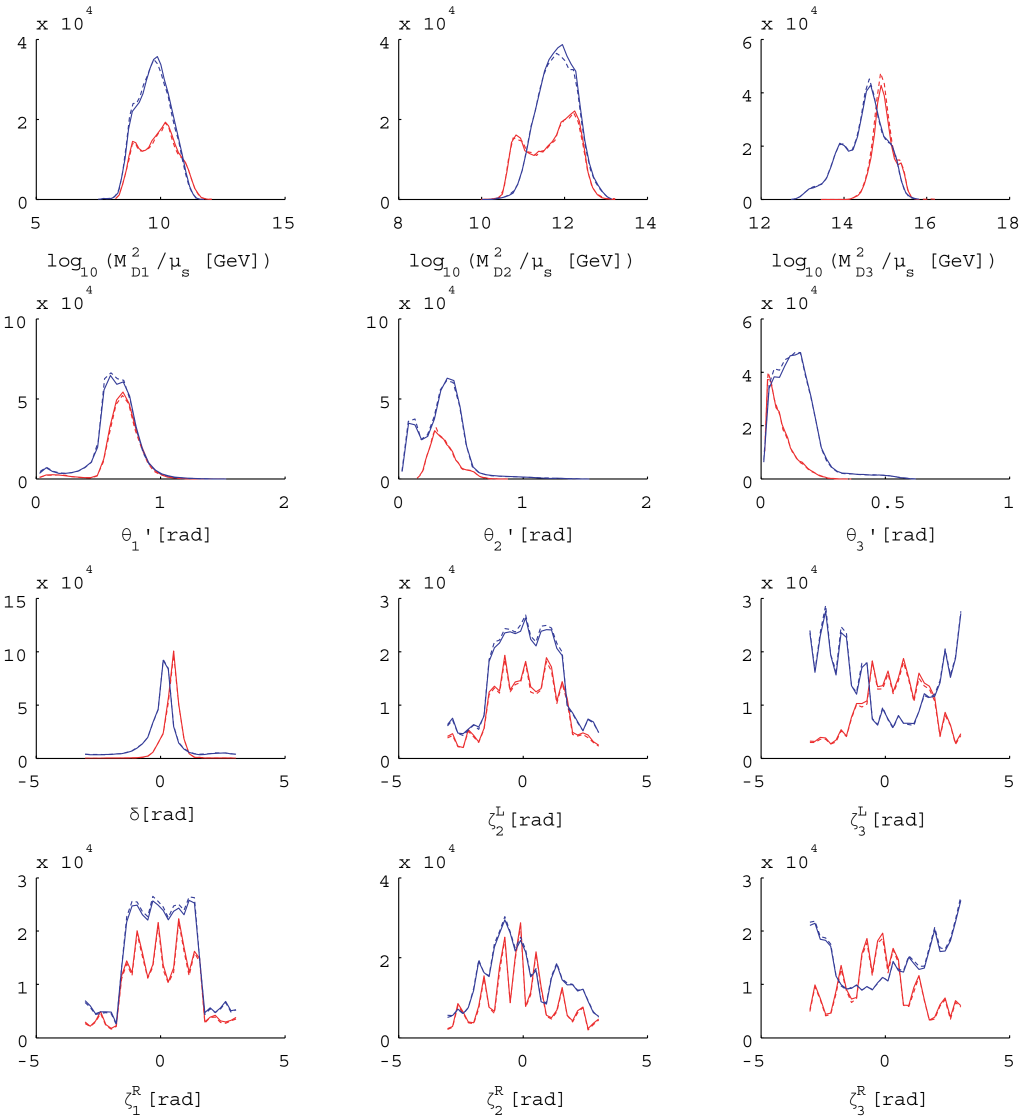

Figure 2: The distributions of the predicted mass ratios, mixing angles and phases for M_D. The signs of each mass eigenvalues are chosen as follows: The red solid line is $(m_u, m_c, m_t) = (+,+,+); (m_d, m_s, m_b) = (+,+,+)$, the red dotted one is $(m_u, m_c, m_t) = (-,+,+); (m_d, m_s, m_b) = (+,+,+)$, the blue solid one is $(m_u, m_c, m_t) = (+,-,+); (m_d, m_s, m_b) = (-,-,+)$ and the blue dotted one is $(m_u, m_c, m_t) = (-,-,+); (m_d, m_s, m_b) = (-,-,+)$.

can not fix σ. So we can obtain the heavy Dirac neutrino mass matrix M_D as a function of μ_s and the three undetermined parameters, σ, two Majorana phases φ_1 and φ_2. For example, for fixed $\mu_s = 1$ [keV] (For the implication of this value, see the remarks below eq. (2.3).) and $\varphi_1 = \varphi_2 = 0$, we get a prediction for the mass spectra of M_D. The dependences on the parameters σ and U_{e3} for fixed $\sigma = \pi$ are depicted in figure 1. These values are allowed by the present experiments [24] and are accessible and testable by the Large Hadron Collider (LHC) at CERN, in which we are able to discover new particles with masses up to $\lesssim 7$ (TeV) [25].

JHEP06(2005)016

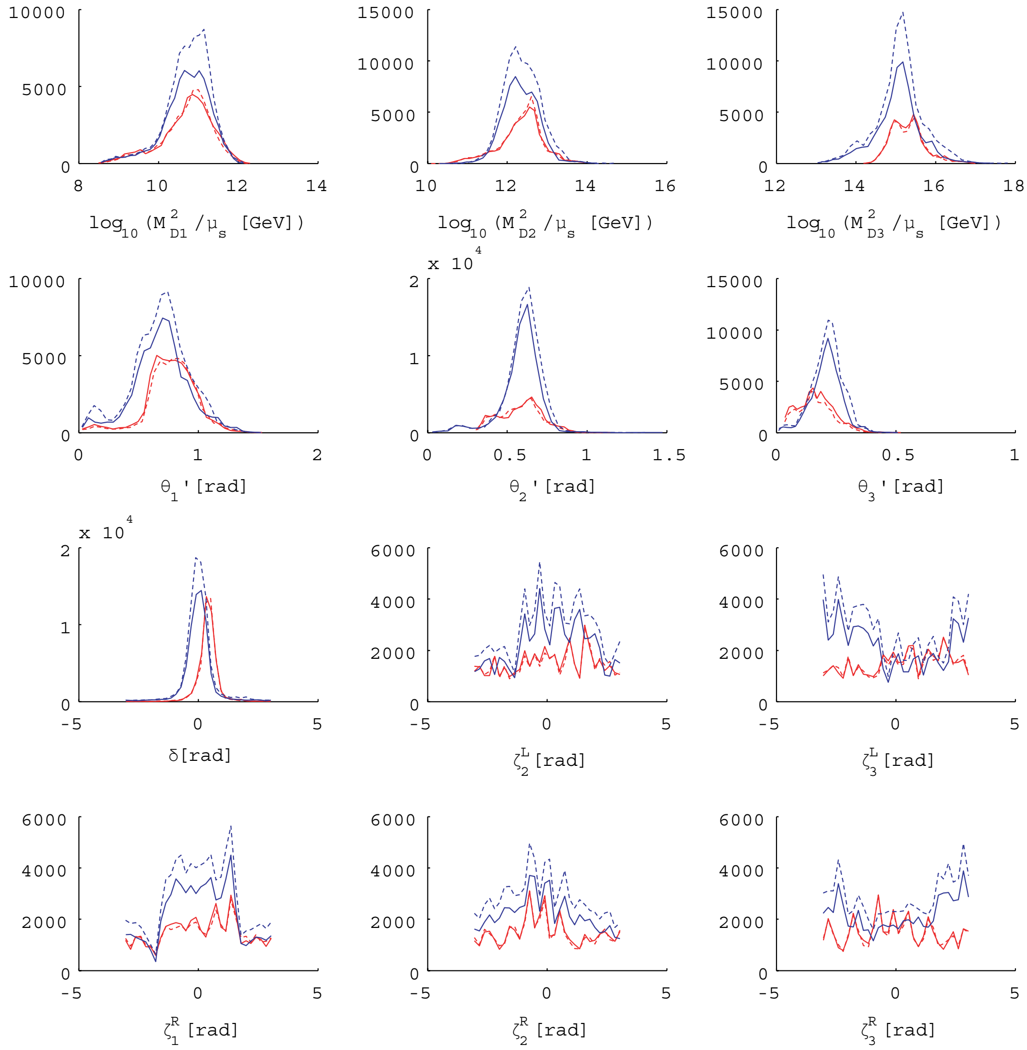

Figure 3: The same distributions as figure 2 but the signs of each mass eigenvalues are chosen as follows: The red solid line is $(m_u, m_c, m_t) = (+,-,+); (m_d, m_s, m_b) = (+,+,+)$, the red dotted one is $(m_u, m_c, m_t) = (-,-,+); (m_d, m_s, m_b) = (+,+,+)$, the blue solid one is $(m_u, m_c, m_t) = (+,+,+); (m_d, m_s, m_b) = (-,-,+)$ and the blue dotted one is $(m_u, m_c, m_t) = (-,+,+); (m_d, m_s, m_b) = (-,-,+)$.

Of course, these values depend on the ambiguous assumptions taken above. We may take another strategy adopted in [21]. As shown in the paper [21], we repeat the substitution of the normally-distributed random numbers which give the experimental values [26]:

$$|m_u\,(2\,\mathrm{GeV})| = 2.9 \pm 0.6\,(\mathrm{MeV})\,, \qquad |m_d\,(2\ \mathrm{GeV})| = 5.2 \pm 0.9\,(\mathrm{MeV})\,, \tag{3.18}$$

$$|m_s\,(2\,\mathrm{GeV})| = 99 \pm 16\,(\mathrm{MeV})\,, \qquad |m_c\,(m_c)| = 1.0 - 1.4\,(\mathrm{GeV})\,, \tag{3.19}$$

$$m_b\,(m_b) = 4.0 - 4.5\,(\mathrm{GeV})\,, \qquad m_t^{\mathrm{direct}} = 174.3 \pm 5.1\,(\mathrm{GeV})\,, \tag{3.20}$$

$$\left|m_e^{\mathrm{pole}}\right| = 0.510998902 \pm 0.000000021\,(\mathrm{MeV})\,, \tag{3.21}$$

JHEP06(2005)016

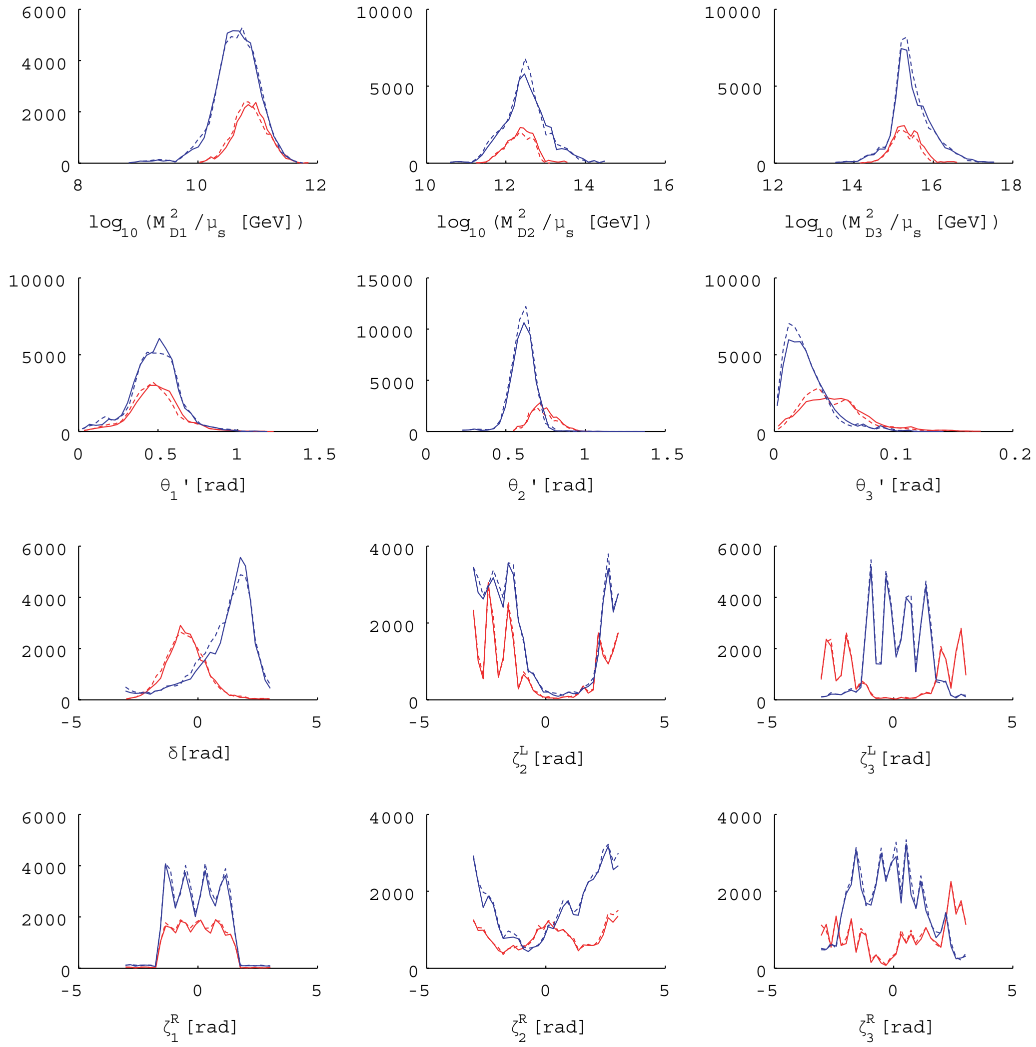

Figure 4: The same distributions as figure 2 but the signs of each mass eigenvalues are chosen as follows: The red solid line is $(m_u, m_c, m_t) = (+,+,+); (m_d, m_s, m_b) = (-,+,+)$, the red dotted one is $(m_u, m_c, m_t) = (-,+,+); (m_d, m_s, m_b) = (-,+,+)$, the blue solid one is $(m_u, m_c, m_t) = (+,-,+); (m_d, m_s, m_b) = (+,-,+)$ and the blue dotted one is $(m_u, m_c, m_t) = (-,-,+); (m_d, m_s, m_b) = (+,-,+)$.

$$\left|m_\mu^{\rm pole}\right| = 105.658357 \pm 0.00005, \qquad m_\tau^{\rm pole} = 1776.99 \pm 0.29 \ ({\rm MeV}), \tag{3.22}$$

$$\sin\theta_{12} = 0.2229 \pm 0.0022, \qquad \sin\theta_{23} = 0.0412 \pm 0.0020, \tag{3.23}$$

$$\sin\theta_{13} = 0.0036 \pm 0.0007, \qquad \delta = (59 \pm 13)^\circ \tag{3.24}$$

for the quark and charged lepton masses and the CKM mixing and the Dirac phase parameters 10,000 times. On the other hand, about the remaining parameters, we assume eq. (3.16), $m_1 = 10^{-3}$ (eV) and $U_{e3} = 0$ at the GUT scale, and Majorana phases and σ move from 0 to 2π in 8 equal intervals. Namely, we scan the possible ranges of undetermined parameters σ, φ_1, φ_2 and plotted the three masses of M_D, the three mixing angles

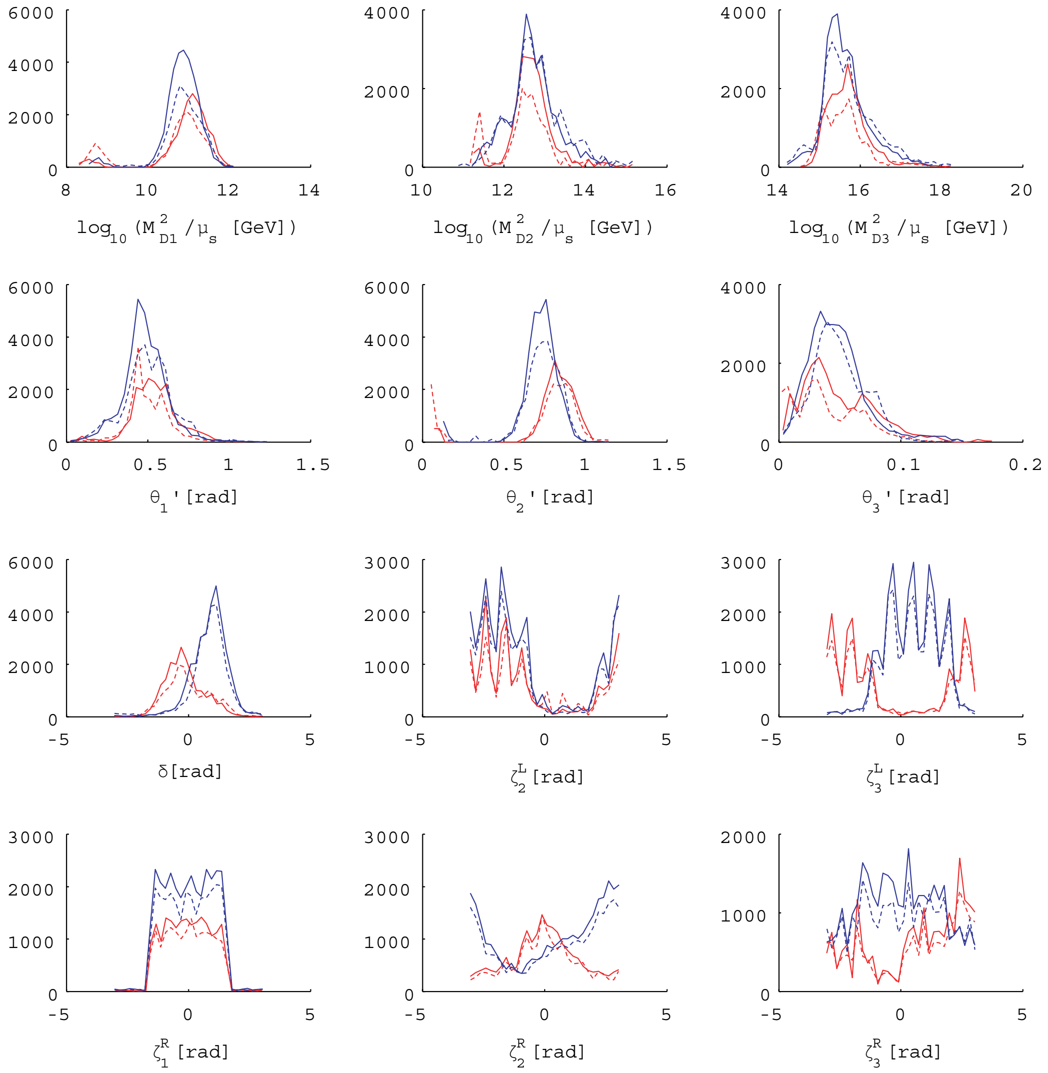

Figure 5: The same distributions as Figure 2 but the signs of each mass eigenvalues are chosen as follows: The red solid line is $(m_u, m_c, m_t) = (+, -, +); (m_d, m_s, m_b) = (-, +, +)$, the red dotted one is $(m_u, m_c, m_t) = (-, -, +); (m_d, m_s, m_b) = (-, +, +)$, the blue solid one is $(m_u, m_c, m_t) = (+, +, +); (m_d, m_s, m_b) = (+, -, +)$ and the blue dotted one is $(m_u, m_c, m_t) = (-, +, +); (m_d, m_s, m_b) = (+, -, +)$.

and five phases of $\mathcal{U}$ which diagonalises the mass matrix M_D in the basis where M_e is real diagonal in figures 2–5. Here we calculated the distributions for sixteen sets of possible combinations of mass signatures of up-type and down-type quarks. figure 1 corresponds to the blue solid line of figure 2 with $\mu_s = 1$ (KeV).

Finally, it is remarkable to say that the see-saw mechanism itself (or the types of it) can never been proofed and all the models should take care of all the types of the see-saw mechanism including the alternatives to it [27, 28]. The test of all these models is due to the applications to the other phenomelogical consequences, for example, the lepton flavour violating processes and so on [29, 30].

4. Summary

In this paper, we have constructed an SO(10) model in which the smallness of the neutrino masses are explained in terms of the type-III see-saw mechanism. To evaluate the parameters related to the singlet neutrinos, we have used the minimal SUSY SO(10) model. This model can simultaneously accommodate all the observed quark-lepton mass matrix data with appropriately fixed free parameters. Especially, the neutrino-Dirac-Yukawa coupling matrix are completely determined. Using this Yukawa coupling matrix, we have calculated the masses and mixings for the not-so-heavy singlet neutrinos. The obtained ranges of the mass of M_D is interesting since they are testable by a forthcoming LHC experiment.

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References

[1] S. Weinberg, *Baryon and lepton nonconserving processes*, *Phys. Rev. Lett.* **43** (1979) 1566.

[2] T. Yanagida, in *Proceedings of the workshop on the unified theory and baryon number in the universe*, O. Sawada and A. Sugamoto eds., KEK, Tsukuba, 1979;
M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, D. Freedman and P. van Niewenhuizen eds., north-Holland, Amsterdam 1979;
R.N. Mohapatra and G. Senjanovi'c, *Neutrino mass and spontaneous parity nonconservation*, *Phys. Rev. Lett.* **44** (1980) 912.

[3] NuTeV collaboration, G.P. Zeller et al., *A precise determination of electroweak parameters in neutrino nucleon scattering*, *Phys. Rev. Lett.* **88** (2002) 091802 [hep-ex/0110059];
The theoretical explanations of this anomaly by including heavy singlet neutrinos are given in [4].

[4] T. Takeuchi, *The NuTev anomaly, neutrino mixing and a heavy Higgs*, hep-ph/0209109;
W. Loinaz, N. Okamura, T. Takeuchi and L.C.R. Wijewardhana, *The NuTev anomaly, neutrino mixing and a heavy Higgs*, *Phys. Rev.* **D 67** (2003) 073012 [hep-ph/0210193];
W. Loinaz, N. Okamura, S. Rayyan, T. Takeuchi and L.C.R. Wijewardhana, *The nutev anomaly, lepton universality and non-universal neutrino-gauge couplings*, *Phys. Rev.* **D 70** (2004) 113004 [hep-ph/0403306].

[5] E. Witten, *New issues in manifolds of* SU(3) *holonomy*, *Nucl. Phys.* **B 268** (1986) 79.

[6] R.N. Mohapatra, *Mechanism for understanding small neutrino mass in superstring theories*, *Phys. Rev. Lett.* **56** (1986) 561.

[7] R.N. Mohapatra and J.W.F. Valle, *Neutrino mass and baryon-number nonconservation in superstring models*, *Phys. Rev.* **D 34** (1986) 1642.

[8] J.W.F. Valle, in *Nuclear beta decays and neutrino: proceedings*, T. Kotani, H. Ejiri, E. Takasugi eds., Singapore, World Scientific, 1986, p. 542.

[9] S.M. Barr, *A different see-saw formula for neutrino masses*, *Phys. Rev. Lett.* **92** (2004) 101601 [hep-ph/0309152].

[10] I. Melo, *The phenomenology of neutral heavy leptons*, hep-ph/9612488.

[11] Z.G. Berezhiani, A.Y. Smirnov and J.W.F. Valle, *Observable majoron emission in neutrinoless double beta decay*, *Phys. Lett.* **B 291** (1992) 99 [hep-ph/9207209].

[12] V. Berezinsky and J.W.F. Valle, *The KeV majoron as a dark matter particle*, *Phys. Lett.* **B 318** (1993) 360 [hep-ph/9309214].

[13] K.S. Babu and R.N. Mohapatra, *Predictive neutrino spectrum in minimal* SO(10) *grand unification*, *Phys. Rev. Lett.* **70** (1993) 2845 [hep-ph/9209215].

[14] K. Matsuda, Y. Koide and T. Fukuyama, *Can the* SO(10) *model with two Higgs doublets reproduce the observed fermion masses?*, *Phys. Rev.* **D 64** (2001) 053015 [hep-ph/0010026].

[15] K. Matsuda, Y. Koide, T. Fukuyama and H. Nishiura, *How far can the* SO(10) *two Higgs model describe the observed neutrino masses and mixings?*, *Phys. Rev.* **D 65** (2002) 033008 [hep-ph/0108202].

[16] T. Fukuyama and N. Okada, *Neutrino oscillation data versus minimal supersymmetric* SO(10) *model*, *JHEP* **11** (2002) 011 [hep-ph/0205066].

[17] H.S. Goh, R.N. Mohapatra and S.P. Ng, *Minimal SUSY* SO(10), *B τ unification and large neutrino mixings*, *Phys. Lett.* **B 570** (2003) 215 [hep-ph/0303055].

[18] H.S. Goh, R.N. Mohapatra and S.P. Ng, *Minimal SUSY* SO(10) *model and predictions for neutrino mixings and leptonic CP-violation*, *Phys. Rev.* **D 68** (2003) 115008 [hep-ph/0308197].

[19] B. Bajc, G. Senjanović and F. Vissani, *B-τ unification and large atmospheric mixing: a case for non-canonical see-saw*, *Phys. Rev. Lett.* **90** (2003) 051802 [hep-ph/0210207].

[20] B. Dutta, Y. Mimura and R.N. Mohapatra, *CKM CP-violation in a minimal* SO(10) *model for neutrinos and its implications*, *Phys. Rev.* **D 69** (2004) 115014 [hep-ph/0402113].

[21] K. Matsuda, *Phenomenological analysis of lepton and quark yukawa couplings in* SO(10) *two Higgs model*, *Phys. Rev.* **D 69** (2004) 113006 [hep-ph/0401154].

[22] K. Matsuda, T. Fukuyama and H. Nishiura, SO(10) *GUT and quark-lepton mass matrices*, *Phys. Rev.* **D 61** (2000) 053001 [hep-ph/9906433].

[23] H. Fusaoka and Y. Koide, *Updated estimate of running quark masses*, *Phys. Rev.* **D 57** (1998) 3986 [hep-ph/9712201].

[24] L3 collaboration, P. Achard et al., *Search for heavy isosinglet neutrino in* e^+e^- *annihilation at Lep*, *Phys. Lett.* **B 517** (2001) 67 [hep-ex/0107014].

[25] S.N. Gninenko, M.M. Kirsanov, N.V. Krasnikov and V.A. Matveev, *Prospects of detecting massive isosinglet neutrino at LHC in the CMS detector*, hep-ph/0301140.

[26] Particle Data Group collaboration, S. Eidelman et al., *Review of particle physics*, *Phys. Lett.* **B 592** (2004) 1.

JHEP06(2005)016

[27] H. Murayama, *Alternatives to seesaw*, *Nucl. Phys.* **137** *(Proc. Suppl.)* (2004) 206 [hep-ph/0410140].

[28] A.Y. Smirnov, *Alternatives to the seesaw mechanism*, hep-ph/0411194.

[29] F. Deppisch and J.W.F. Valle, *Enhanced lepton flavour violation in the supersymmetric inverse seesaw model*, hep-ph/0406040.

[30] A. Ilakovac and A. Pilaftsis, *Flavor violating charged lepton decays in seesaw-type models*, *Nucl. Phys.* **B 437** (1995) 491 [hep-ph/9403398].